Extended Abstract

Kinetic study of laboratorial batch sieving

Statistical validation of kinetic models and establishment of a sieving time scale

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Abstract

This work was divided in two parts, one theoretical and another laboratorial, with the purpose of answering one of the two questions that come to the surface when we perform batch sieving: what the ideal time interval of the operation is for maximum efficiency. Sieving laboratory tests were performed using samples of different material masses, different granulometry compositions and different times of operation. The samples were composed by three fractions, an oversize fraction and two undersize fractions. To determine the recovery of the two undersize fractions in the undersize product, sieving tests were performed. On a primary phase, the recovery of the undersize fraction was determined, and on a secondary phase, the recovery of each of the two portions of undersize in the undersize product was determined.

Five kinetic models were tested, with the identification/selection of the best one. The set of tested models includes the original Trumic and Magdalinovic model, and a new modified version where a new parameter was added, designated by α, to obtain a significant improvement in the quality of the adjustments given by the modified model; the Standish model; a variant of the Standish model; the Ferrara and Preti model, with the inclusion of recovery limits, and the same model, but with a new combination of the two existing conditions, crowded and separate sieving. The evaluation and validation process allowed to identify the modified Trumic and Magdalinovic model, as well as the Ferrara and Preti model with the new combination of the two conditions, as the best models to better describe the time evolution of the laboratorial sieving process.

Keywords: sieving, kinetics, kinetic models

Introduction

Test sieving is the most widely used method for particle size analysis, covering a large range of particle sizes. The process occurs by passing a sample of known weight of material through a series of sieves of different aperture sizes and weighing the amount collected on each sieve, to determine the percentage weight in each size fraction. Normally, the sieves are shaken to expose the particles to the openings. This process can happen with addition of water or without it (Will's, et all, 2016). It is necessary to guarantee that the process of sieving continues until it is possible to guarantee that all particles have passed through the openings (Gupta et al., 1974).

Although sieving is a very known and used method, its kinetic aspects have been neglected over the years, even though these aspects dictate if the sieving process was efficient or not (Yekeler, et al., 2014).

Many factors influence the sieving efficiency, such as the sample size, sieving time, and others. Therefore, it is of importance to consider the right time scale and right sample size to obtain the best recovery numbers (Will's et al., 2016).

Kinetic Models

Although there are various articles and studies done about industrial sieving, there are very few mathematical models for laboratorial sieving, especially sieving with the help of magnetic agitators (Trumic et al., 2010).

The parameter estimation problem is tightly connected to the development of mathematical models. To said problem is normally associated a step in which the validation the respective model is performed, based on statistical testing about the residues between the obtained and predicted responses of the model.

A model is considered valid if:

1. It explains the biggest fraction possible of the data variability;
2. It is a simple model, which means it is a model with reduced number of parameters.

In a non-linear parameter estimation issue, the models are usually presented in their reduced standard form, such as

$$y = f(x, \theta)$$

Where y is the dependent variable, x the independent variables, θ the adjustment parameters and f the mathematical relations of the model.

Since, in general, the number of observations measured exceeds the number of parameters to estimate, it isn't possible to determine a single set of values for the parameters that satisfies all equations and the deviations of the measured and predicted values. It is then necessary to add a new criterion, the objective function, to get a single set of values for the parameters, given by:

$$z = f(x, \theta) + e$$

With z being the dependent variable measured and e the experimental error, assuming that the model is adequate.

A model is adequate if we can explain the residues of said adjusted model as being errors in the observations. This means that the considered hypothesis is that the observation errors are events of random variables of mean 0 and covariance matrixes $Covar(\theta)$.

Five models were tested in this work: Standish' model, a variant of said model, Trumic and Magdalinovic's model, a modified version of said model that included the addition of a new par and Ferrara and Preti's model, with an added screening type, a combination of crowded and separate screening.

All models, except for the Ferrara and Preti model, were models with three parameters to be estimated. Ferrara and Preti had four.

To the original Trumic and Magdalinovic model, one without linearization, was added a constant α, to obtain a significant improvement in the quality of the adjustments provided by the model. The variation velocity of the kinetic constant as a function of the granulometric distribution of the material on the sieve is determined, in its original version, by $\frac{m_t}{m_0}$, which means that it is a linear variation. A nonlinear relation, given by $\left(\frac{m_t}{m_0}\right)^\alpha$, allows us to adjust the variation velocity of the kinetic constant, speeding itu p when $\alpha > 1$, or slowing it down, when $\alpha < 1$.

The combined crowded and separate screening regime of the Ferrara and Preti model established that during the first ten seconds of sieving, the screening would be considered crowded. After ten seconds, due to most of the material having fallen, it was separate screening.

Methodology

Initially, preparation of samples needed to be done, to get the process started. Nine samples were mixed, containing three different granulometry sizes of sand: a coarser size, an intermedium size, and a finer size.

Since the two screening conditions, crowded and separate, were intended for the study, calculations were made to have three sets of three samples each, where the two first were in crowded screening and the last one was separate screening.

There were three different compositions of the samples, that were repeated for each set of three samples. The coarser particle size remained the same for all samples, with the two types of finer size, I_1 being the intermediate size and I_2 the finer size, being varied between equal amounts, superior and inferior amounts for each of the two.

S, the coarser size, was sand of caliber -2.000 +1.400 mm; I_1, the intermediate size, was sand of caliber -1.000 +0.710 mm and I_2, the finer size, was sand of caliber -0.710 +0.500 mm.

Table 1. Composition of all samples

Sample	**Composition (%)**			**Total Weight (g)**
	S	***I_1***	***I_2***	
1	50	25	25	300
2	50	15	35	300
3	50	35	15	300
4	50	25	25	200
5	50	15	35	200
6	50	35	15	200
7	50	25	25	100
8	50	15	35	100
9	50	35	15	100

After the composition of all samples, seen on table 1, the sieving process occurred. Before proceeding to the final process of sieving, preliminary tests were performed, to establish the best combination of sample sizes and sieving time. The sieving times chosen for these tests started at 30 seconds, followed by sieving during 1, 2, 4 and 8 minutes.

After those preliminary sets, where each sample of the three sets was sieved through different time stamps, the undersize product being removed after each time stamp, two sets of tests were performed.

Those last two sets kept the same procedure as before, with the material being placed in the agitator and being sieved for each time stamp chosen, undersize removed after each one, and repeated the process for the next time stamp.

To avoid early loss of undersize product before the machine could start the sieving process, a paper filter was used to place the material on the sieve, only removed when the process was about to begin.

Results and Discussion

In this chapter, the results of the laboratorial part of the work are presented, for all the tests performed.

For the preliminary tests, it was noted that most of the undersize material was recovered after the first 30 seconds of sieving, as seen on figure 1. The remaining sieving results were small increments of undersize that was trapped between the coarser particles. It can also be seen that the recovery of all time scales is very close to 100% by the time the 8 minutes were reached.

Although it wasn't a significant loss, it is important to note that, due to material loss, sample 1 didn't have the full 300g of total weight, but only 299,6g.

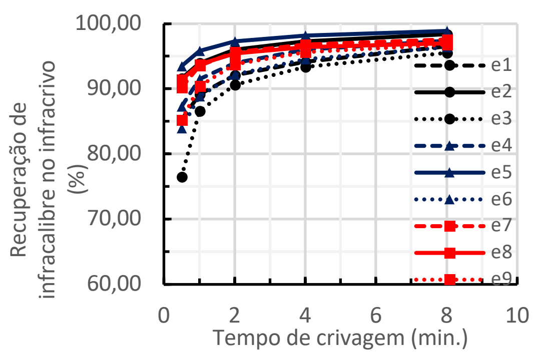

Figure 1. Recovery of the undersize of the sieving product for each test, as a function of the time of sieving for the preliminary tests

For the first set of tests, the time scale was changed. Two new time stamps were added, and one was removed. The samples were then sieved for 10, 20 and 30 seconds, and 1, 2 and 4 minutes. This change was done in order to obtain better recovery results, distributed over the small increases of time.

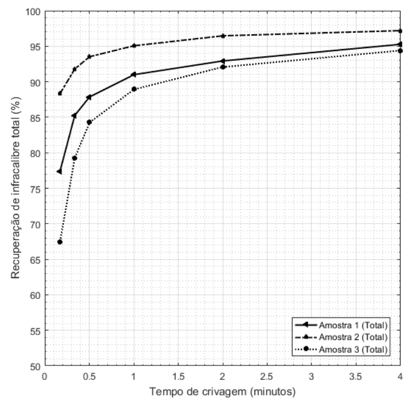

Figure 2. Recovery of the total undersize as a function of sieving time for a total weight of 300g

An improvement was observed, compared to the preliminary tests, however, there was still the issue of 88 to 95% of the undersize material being recovered after only 1 minute of sieving, as seen on figures 2, 3 and 4. It was also noted that the smaller the sample, the bigger the recovery, which was to be expected considering that there is less obstacles for the fine particles to reach the sieve openings, as figure 3 and 4 show.

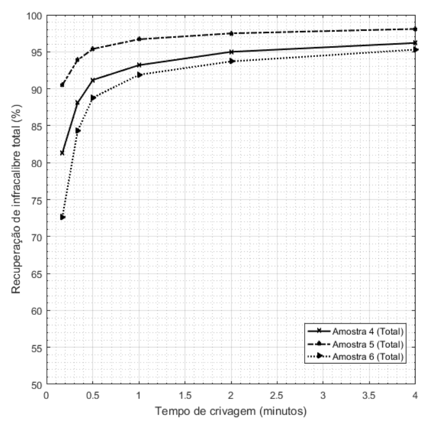

Figure 3. Recovery of the total undersize as a function of sieving time for a total weight of 200g

Another thing to be noted is that the samples with compositions of lesser I_1 and bigger quantity of I_2 obtained better recovery results, which was also expected considering there are less particles of bigger size preventing the smaller particles of reaching the sieve apertures.

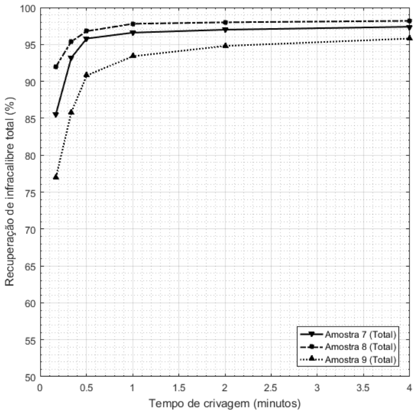

Figure 4. Recovery of the total undersize as a function of sieving time for a total weight of 100g

The second and final set of tests was performed to analyze the product of sieving and understand what quantity of each of the two undersize was recovered.

To achieve this, each of the sieving products was once again sieved, this time for only one period of 5 minutes, to separate the two finer sizes.

Much like the other sets of tests, the samples with the bigger weight demonstrated to have lesser recoveries of I_1 in comparison with the samples that had the smaller weight, which was, once again, to be expected.

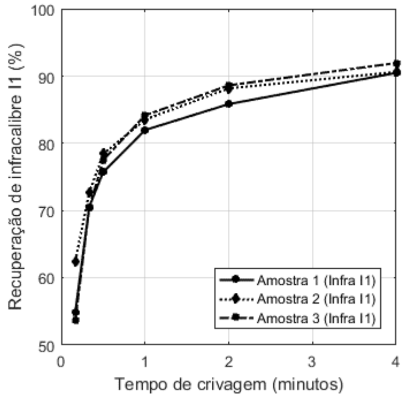

Figure 5. Recovery of undersize 1 of the sieving products in the tests 1,2 and 3, as a function of sieving time, for the second set of tests

The samples with the smallest amount of I_1 were the ones to show bigger recoveries of this material, as was to be expected, and seen on figure 5.

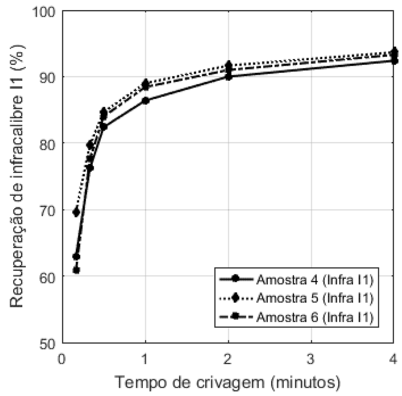

Figure 6. Recovery of undersize 1 of the sieving products in the tests 4,5 and 6, as a function of sieving time, for the second set of tests

All samples showed that, after two minutes of sieving, the recovery values seemed to stabilize, showing that at 4 minutes only vestigial particles pass through the sieve, and all the remaining material still trapped on the sieve surface wasn't going to pass, shown on figures 5,6 and 7.

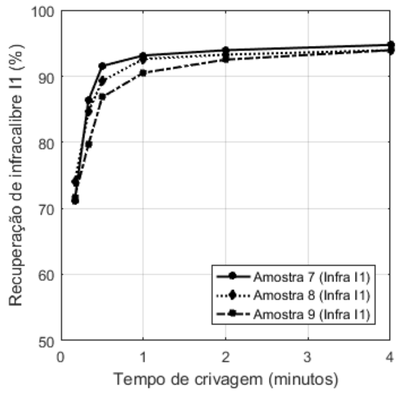

Figure 7. Recovery of undersize 1 of the sieving products in the tests 7,8 and 9, as a function of sieving time, for the second set of tests

Some of the material recovered exceeded the weight included of that size. This phenomenon can be explained by the presence of friction between particles, causing some coarser ones to break and pass through the openings. The friction between particles, however, was not a factor included in this study.

Validation and Selection of the Kinetic Model

Finalized the laboratorial portion of the study, it was then time to analyze the data obtained through it and validate the five different kinetic models considered for this work, in order to select one.

The chosen model's purpose would be to permit investigating and calculating of the variables which had the biggest influence in the results, as well as give suggestions for the most appropriate time scale for sieving with optimized results.

Each model was tested using the *MATLAB*™ tool.

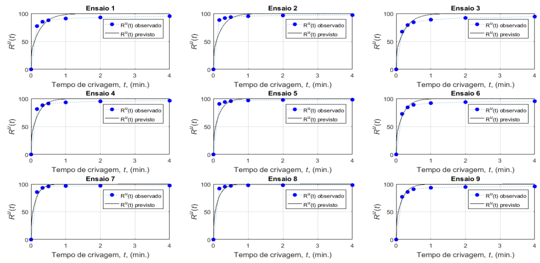

Figure 8. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Standish model

The first model tested was the Standish model, using the following equation.

$$R^u(t) = \sum_{i=1}^{n} \frac{m_{i,0}}{m_0} \times R^u{}_i(t) = \sum_{i=1}^{n} \frac{m_{i,0}}{m_0} \times (1 - \exp(-k_i \times t)) \ (Eq. 1)$$

The results were not what would be expected of a perfect model, considering that the relation between the observed and predicted recoveries was not ideal.

The Akaike criterion valid, since it was below 50, much like constant k, whose values were high as desired. The SQR, however, presented higher values than what was wanted for a valid model. 95% of recovery were obtained before 1 minute of sieving, which is a positive trait.

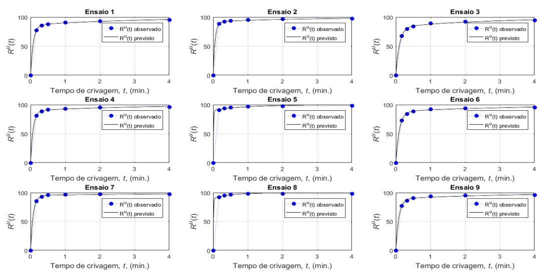

Figure 9. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the variant of the Standish model

For the variant of the Standish model, the results were much better than the original model, using the following equation.

$$R^u(t) = \sum_{i=1}^{m} f_i \times R^u{}_i(t) = \sum_{i=1}^{m} f_i \times (1 - \exp(-k_i \times t)) \ (Eq. 2)$$

The relation between the two recoveries was nearly perfect, which was what was desired for a perfect model.

The Akaike criteria showed lower values compared to the previous model, much like the SQR, two positive points for the model. However, k showed lower values than considered valid, showing poor quality of sieving.

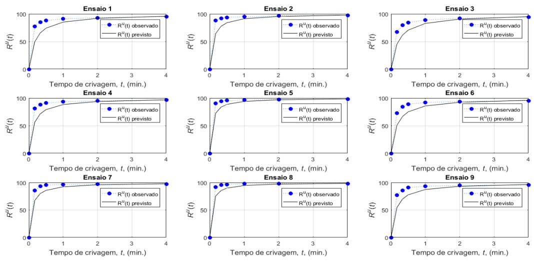

Figure 10. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Trumic and Magdalinovic model, with linear regression

Trumic and Magdalinovic's model, in its original form, had the estimation of k with linear regression and with nonlinear regression. For the first one, the results were behind what was desired, with the relation between the two recoveries being very poor, and the results were obtained using the following equation.

$$y(t) = k \times t \qquad (Eq. 3)$$

The Akaike criteria was not used for this method, since it is a parameter only used for nonlinear models. However, the values of k were considered low, as well as the ones for t_{95}. SQR, however, showed high results, going against what was wanted.

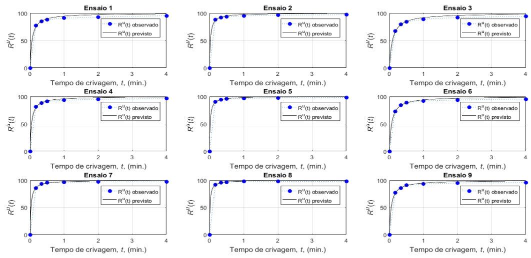

Figure 11. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Trumic and Magdalinovic model, with nonlinear regression

The nonlinear regression results, using the equation bellow, showed great improvement, with the relation between recoveries being far better than the previous.

$$R^u(t) = 1 - \frac{m(t)}{m_0} = 1 - \frac{1}{kt + 1} = \frac{kt}{1 + kt} \quad (Eq. 4)$$

k values were higher than the linear regression ones, showing a high-quality sieving process, with both Akaike criteria and SQR showing much lower results than previously.

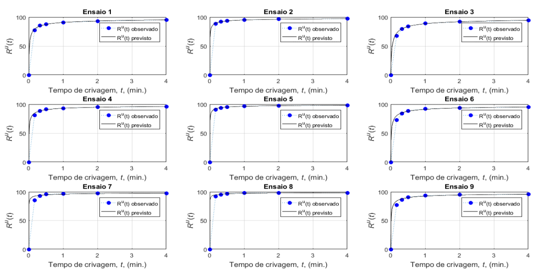

Figure 12. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the modified Trumic and Magdalinovic model

With the modified version of the Trumic and Magdalinovic model, with an added parameter designated by α, the results were very good, and acquired through the following equation.

$$R^u(t) = \left(1 - \frac{m(t)}{m_0}\right) = 1 - \frac{1}{(\alpha \times k \times t + 1)^{\frac{1}{\alpha}}} \quad (Eq. 5)$$

The relation between the recoveries showed, once again, good results. Even so, the k values were much higher than before, showing once again the quality of the sieving process. The Akaike criterion showed low values, as well as SQR. α took the value of 2, which was what offered the best results for the model.

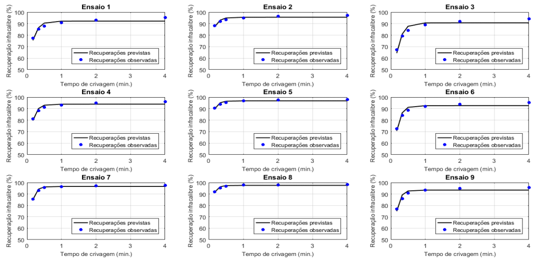

Figure 13. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Ferrara and Preti model, in crowded sieving conditions

Lastly, the Ferrara and Preti model was the final one to be validated. Three different screening conditions were considered: crowded, separate and crowded+separate. For the crowded and separate sieving, a recovery limit was input, to be able to obtain the graphical results of the model. Equations for the crowded and separate conditions can be seen bellow, respectively.

$$G(R_i^u(t)) = 1 - \exp\left(\frac{-\left(\sum_{j=i_s}^{n} \frac{m_j(0)}{\chi_{j,i}}\left[\left(1-R_i^u(t)\right)^{\chi_{j,i}} - 1\right] + \frac{Kc_i}{M(0)}t\right)}{\sum_{j=1}^{i_s-1} m_j(0)}\right) \quad (Eq.6)$$

$$\left(1 - R_i^u(t)\right) = e^{-Ks_i t} \quad (Eq.7)$$

For the crowded screening, the relation between the two recoveries, although not perfect, is considered good. Observing the results of the parameters mentioned along the validation of models, however, it is shown that the Akaike criterion is quite high, going against what is required, and k shows relatively low values, showing a poor-quality sieving process. SQR, however, shows low values as needed.

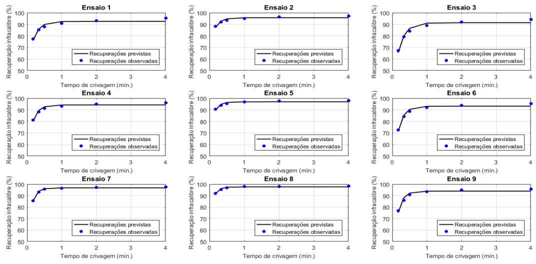

Figure 14. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Ferrara and Preti model, in separate screening conditions

The separate screening shows a relation between the two recoveries as being quite similar to the previous one. What shows changes are the parameters, like the Akaike criteria, that shows much higher values, against what is required, k shows high values, confirming a high-quality sieving process. SQR values showed lower values when compared to the previous conditions.

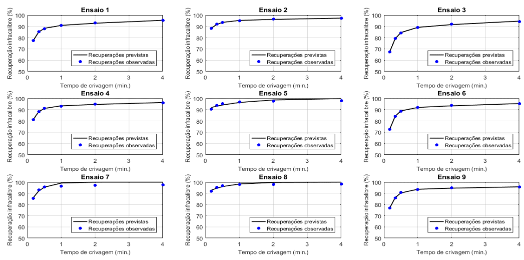

Figure 15. Predicted Recovery curves vs Observed recovery curves of the undersize of the sieving product as a function of sieving time, for the Ferrara and Preti model, for the combination of crowded+separate screening conditions

The last screening condition, one created for this work, shows the best relation between observed and predicted recoveries. Backing up the wonderful results seen for the recoveries, are the parameters estimated. k admitted quite high results, like desired. SQR also shows good, low results, much like the Akaike criteria, which had the best results of all conditions tested.

Due to these parameters and relation between the curves of recoveries, the model to be chosen was the crowded+separate regime of the Ferrara and Preti model, as it was the only one who could perfectly describe the evolution of the sieving process through the time scale chosen.

It was possible to realize that, except for the Ferrara and Preti model, the variation pattern of the velocity constants, k, is similar throughout the models. There is a decrease of k with the increase of the percentage of the critical fraction I_1, and an increase of k with the decrease of the sample size. An example of this can be seen on figure 16.

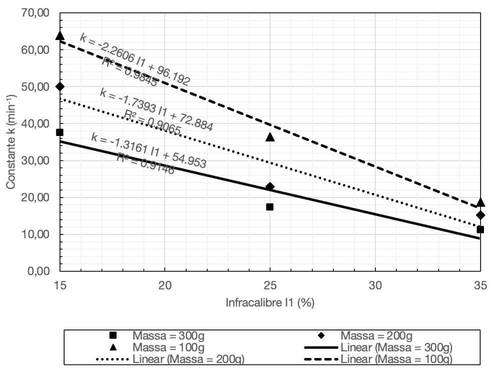

Figure 16. Variation of k of the original Trumic and Magdalinovic model (non-linear) with the weight and composition of samples

For the Ferrara and Preti model with crowded screening, the velocity constant kc increases when I_1 increases. For the separate screening, the velocity constant ks varies similarly, increasing with the decrease of weight, and a significant decrease with the increase of I_1. An example of this can be seen on figure 17.

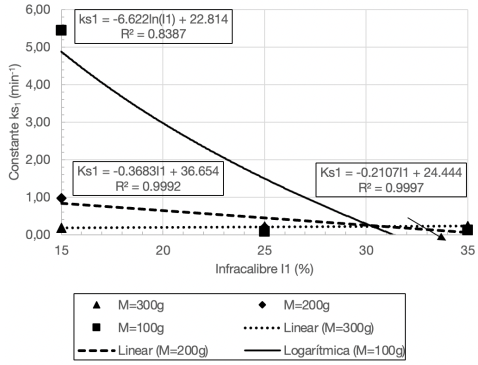

Figure 17. Variation of ks of the Ferrara and Preti model for separate screening conditions with the weight and composition of samples

Conclusions

For the first set of tests, for all samples, the biggest recovery obtained was at only 10 seconds of sieving. The oversize material showed no significant changes from the beginning of the sieving process to the end, having small increases that could be explained by the blinding of apertures of the sieve, whose particles came out when the oversize was removed, or by some undersize particles being trapped between larger ones. This can be proven by the small decrease of total weight in the undersize product.

The second set of tests noted that all the recovery products were high, meaning that almost all material of undersize was recovered. It was also possible to see that the biggest recoveries were obtained for the samples where the I_2 material fraction was higher than the I_1 material.

Observing all sets of tests and their results, it is possible to conclude that the biggest recoveries were obtained for the samples in which the total weight was small, meaning the 100g samples. The best time scale, as was also noted throughout all sets of tests, was given by each of the kinetic models tested, and was considered as the best for the model chosen, the Ferrara and Preti model for the crowded+separate conditions.

It was also possible to provide modifications to existing models, improving them to possibly be valid for the results obtained in the laboratorial work. These modifications included adding a new parameter to the Trumic and Magdalinovic model to improve the quality of adjustments given by it, and the imposition of a new condition of screening in the Ferrara and Preti model, that offers a much better solution than the two regimes separately.

Both the Modified Trumic and Magdalinovic model and the Ferrara and Preti model in the new screening conditions proved to be the most efficient models to describe the evolution of the sieving process over time, proving that the modification performed in this study improved the quality of the adjustments of the model, even though it was at the cost of adding more parameters to the models.

The Ferrara and Preti model in the new screening conditions proved to be the most adequate for the present dissertation, since between it and the Modified Trumic and Magdalinovic model, it had the best SQR values, as well as a residue mean of 0.

Choosing the Ferrara and Preti model as the most adequate one shows that it is possible to obtain adjustments of better quality, at the cost of a more complex model, since contrary to what was seen for the remaining models for this work, this model has four parameters. The bigger the quantity of parameters, the more complex the model becomes.

Although this time scale has been proven to work for sand, it is still unsure if it will work for other materials. It would be interesting to proceed with the study of the time scale with other materials, or even with other time scales, going as low as starting with 5 seconds sieving.

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